Figure 5-1: Spherical coordinate system for a wave traveling in the $\hat{k}$ direction.
Figure 5-2: Polarization ellipse in the v–h plane for a wave traveling in the \( \hat{k} \) direction.
Figure 5-3: EM beam incident upon an object is scattered along many directions.
Figure 5-4: The duplexer used by a monostatic radar allows the antenna to be shared between the transmitter and the receiver.
Figure 5-5: Coordinate systems and scattering geometry for (a) the forward scattering alignment convention and (b) the backscatter alignment (BSA) convention.
**Figure 5-6:** A vertically polarized incident wave $E^i_v$, defined at the location of the scattering object, generates a scattered wave with $E^s$ defined at range $R_r$ from the object. A vertically polarized receiving antenna measures $E^s_v = (e^{-jkR_r/R_r}) \tilde{S}_{vv} E^i_v$. 
Figure 5-7: Power density $S^i_q$ of the incident wave is defined at the location of the target, whereas $S^s_p$ of the scattered energy is defined at the location of the receive antenna.
Figure 5-8: Illumination geometry for a distributed target.
Figure 5-9: Real-aperture radar (RAR) geometry and resolved surface area.
Along-track resolution

Real-aperture \( r_a = \frac{\lambda R}{l} \)

Synthetic-aperture \( r_a = \frac{l}{2} \)

Figure 5-10: An illustration of how synthetic aperture works.
Figure 5-11: Measured azimuthal pattern of the radar backscatter from a B-26 airplane at 10 cm wavelength [from Ridenour, 1947, courtesy McGraw-Hill Book Company].
Figure 5-12: The sketch in (a) shows how the measurements [shown in (b)] of the backscatter from an asphalt surface were acquired. The incidence angle was 40°, the frequency 35 GHz, the platform height 10.4 m, and the polarization vv [from Ulaby et al., 1988a].
Figure 5-13: Image speckle refers to the pixel-to-pixel variation of image tone. This is a $1\,\text{m} \times 1\,\text{m}$ resolution Ku-band SAR image of an agricultural area [Sandia National Lab].
Random Variables

For any random variable \( x \), defined over the range \( x_1 \) to \( x_2 \) and characterized by a probability density function (pdf) \( p(x) \):

\[
\bar{x} = \langle x \rangle = \int_{x_1}^{x_2} x \ p(x) \ dx = \text{mean value of } x
\]

\[
\overline{x^2} = \langle x^2 \rangle = \int_{x_1}^{x_2} x^2 \ p(x) \ dx = \text{second moment of } x
\]

\[
s_x^2 = \langle x^2 \rangle - \langle x \rangle^2 = \text{variance of } x
\]

\( s_x = \text{standard deviation of } x \)

\[
\beta_x = \left( \frac{s_x}{\bar{x}} \right)^2 = \text{normalized variance of } x
\]

\[
P(x \leq x') = \int_{x_1}^{x'} p(x) \ dx
\]

= cumulative distribution of \( x \leq x' \)
Figure 5-14: The illuminated area $A$ contains $N_s$ randomly distributed scatterers.
Figure 5-15: The vector $\mathbf{E} = E_e e^{j\phi}$ is the phasor sum of $N_s$ fields.
Figure 5-16: Plot of (a) probability density functions and (b) cumulative distributions for $f$ and $F$. 
Fading random variable $F$

$p(F) \cdot \Delta F$

$e^{-F} \cdot \Delta F$

Figure 5-17: The measured pdf of the backscatter from asphalt [corresponding to the data in Fig. 5-12(b)] is found to be in good agreement with the exponential pdf based on the Rayleigh fading model.
Figure 5-18: Probability density function for $N = 1, 4,$ and 10 for (a) $f_N$ (linear deduction), (b) $F_N$ (square-law detection), and (c) $g_N$ (square-root intensity).
Figure 5-19: Plots of the 5% and 95% cumulative distribution levels versus $N$. The vertical spacing between these two curves is a measure of the “confidence interval” associated with a measurement of the radar backscatter.
Figure 5-20: Variation of $s_{fN}/\overline{f_N}$ (amplitude image), $s_{FN}/\overline{F_N}$ (intensity image), and $s_{gN}/\overline{g_N}$ (square-root intensity image) as a function of the number of looks (independent samples) $N$. 
Figure 5-21: Simulated amplitude images of three distributed targets: trees with $\sigma^0_t = 36 \text{ m}^2/\text{m}^2$, grass with $\sigma^0_g = 16 \text{ m}^2/\text{m}^2$, and soil with $\sigma^0_s = 4 \text{ m}^2/\text{m}^2$. Averaging multiple pixels trades off spatial resolution for improved radiometric resolution (less pixel-to-pixel variation).
Figure 5-22: As the number of independent samples $N$ is increased from 1 to 16, the confidence intervals around the means decrease by $1/\sqrt{N}$.
Figure 5-23: The four-image set provides a visual illustration of the relationships between image speckle, number of looks $N$, spatial resolution, and interpretability.
Figure 5-24: Histograms of image tone $T$ (digital number) of square-root-intensity Seasat SAR images of three categories of distributed targets. The water category provides a good fit to the theoretical pdf given by Eq. (5.90); the forest category exhibits a significant departure due to textural variations and the urban category exhibits the greatest departure from theory. Urban scenes often have bright, dominant scatterers, which violates Assumption 6 of the Rayleigh model.
Figure 5-25: The pixel-to-pixel variation in the cornfield image is almost entirely due to Rayleigh fading (speckle), whereas the forest-parcel image includes spatial patterns (in addition to speckle) associated with the density variation of trees.
Figure 5-26: The 3 m × 3 m resolution 2-look image in (a) was subjected to six different despeckling filters [courtesy of Oliver and Quegan].
Figure 5-27: Random, isotropic surface $z(x, y)$: (a) pictorial view, (b) measured height profile $z(x)$, (c) pdf of digitized height profile, and (d) autocorrelation function $\rho(\xi)$, where $\xi$ is the displacement between two points on the surface.
Gaussian
\[ \rho_G(\xi) = \exp\left\{-\frac{\xi^2}{l^2}\right\} \]

Exponential
\[ \rho_e(\xi) = \exp\left\{-\frac{|\xi|}{l}\right\} \]

Figure 5-28: Exponential and Gaussian correlation functions.
Figure 5-29: Bistatic-scattering coordinate system.
The bistatic-scattering pattern consists of a coherent component along the specular direction and a noncoherent component along all directions. For a perfectly smooth surface, only the coherent component exists, and at the opposite extreme, for a very rough surface the coherent component becomes negligibly small in comparison with the noncoherent component.

Figure 5-30: The bistatic-scattering pattern consists of a coherent component along the specular direction and a noncoherent component along all directions. For a perfectly smooth surface, only the coherent component exists, and at the opposite extreme, for a very rough surface the coherent component becomes negligibly small in comparison with the noncoherent component.
Figure 5-31: Measured and calculated reflectivities for h and v polarizations for four surfaces with different EM roughnesses [De Roo and Ulaby, 1994].
Figure 5-32: Variation of (a) $\Gamma_{coh}/\Gamma$ with $\psi = ks \cos \theta$ and (b) $\Gamma_{coh}^v$ with $\theta_i$ for various values of $ks$ [De Roo and Ulaby, 1994].
Figure 5-33: Measured bistatic-scattering pattern for a sand surface with $k_s = 0.73$ at 35 GHz [Ulaby et al., 1988d].
Figure 5-34: Measured bistatic-scattering patterns for a rough surface: (a) $\sigma_{vv}^0$ and (b) $\chi$ [Nashashibi and Ulaby, 2007].
$f = 1.5 \text{ GHz}$
hh polarization

$\sigma_{hh}^0 \approx \sigma_{hh}^{0\text{coh}}$

$\sigma_{hh}^0 \approx \sigma_{hh}^{0\text{inc}}$

Retrieved true $\sigma^0(\theta)$

ks = 0.3
kl = 5
$\varepsilon = 20.9 - j2.8$
$\beta_c = 0.51^\circ$

Figure 5-35: Measured and retrieved backscattering angular pattern for a surface with $ks = 0.3$ at 1.5 GHz [Ulaby et al., 1983].
A polarimetric radar is implemented by alternately transmitting signals out of horizontally and vertically polarized antennas and receiving at both polarizations simultaneously. Two pulses are needed to measure all the elements in the scattering matrix [van Zyl and Kim, 2011].

**Figure 5-36:** Calibration of polarimetric radar.
<table>
<thead>
<tr>
<th>Target</th>
<th>Geometry</th>
<th>Co-pol response</th>
<th>Cross-pol response</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Sphere</td>
<td><img src="image" alt="Sphere" /></td>
<td><img src="image" alt="Co-pol Sphere" /></td>
<td><img src="image" alt="Cross-pol Sphere" /></td>
</tr>
<tr>
<td>(b) Dihedral</td>
<td><img src="image" alt="Dihedral" /></td>
<td><img src="image" alt="Co-pol Dihedral" /></td>
<td><img src="image" alt="Cross-pol Dihedral" /></td>
</tr>
<tr>
<td>(c) Trihedral</td>
<td><img src="image" alt="Trihedral" /></td>
<td><img src="image" alt="Co-pol Trihedral" /></td>
<td><img src="image" alt="Cross-pol Trihedral" /></td>
</tr>
<tr>
<td>(d) Thin horizontal cylinder</td>
<td><img src="image" alt="Thin Horizontal Cylinder" /></td>
<td><img src="image" alt="Co-pol Thin Horizontal Cylinder" /></td>
<td><img src="image" alt="Cross-pol Thin Horizontal Cylinder" /></td>
</tr>
<tr>
<td>(e) Thin 45°-inclined cylinder</td>
<td><img src="image" alt="Thin 45°-Inclined Cylinder" /></td>
<td><img src="image" alt="Co-pol Thin 45°-Inclined Cylinder" /></td>
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</tr>
<tr>
<td>(f) Thin vertical cylinder</td>
<td><img src="image" alt="Thin Vertical Cylinder" /></td>
<td><img src="image" alt="Co-pol Thin Vertical Cylinder" /></td>
<td><img src="image" alt="Cross-pol Thin Vertical Cylinder" /></td>
</tr>
</tbody>
</table>

**Figure 5-37:** Co-pol and cross-pol responses of 6 point targets. LR stands for a left-hand (receive)/right-hand (transmit) polarization configuration.
Figure 5-38: This series of L-band images of San Francisco were synthesized from a single polarimetric image acquired by the NASA/JPL AIRSAR system. The nine images include various combinations of co-polarized (transmit and receive polarizations are the same) and cross-polarized (transmit and receive polarizations are orthogonal) images. Note the relative change in brightness between the city of San Francisco, the ocean, and the Golden Gate Park, which is the large rectangle about 1/3 from the bottom of the images. The radar illumination is from the left.
Figure 5-39: This series of L-band images of the Florida Coast were synthesized from a single polarimetric image acquired by the NASA/JPL UAVSAR system. Radar illumination is from the left. The large body of water is part of Choctawhatchee Bay. See the text for additional description of the area.
Figure 5-40: Measured histograms of the real and imaginary parts of $S_{vv}$ for a soil surface observed by a 4.75 GHz radar scatterometer [Sarabandi, 1992].
Figure 5-41: Measured histograms of the cross-pol phase difference $\phi_x$ and the co-pol phase difference $\phi_c$ of a slightly rough soil surface [Sarabandi, 1992].
\[ \phi_c = \phi_{hh} - \phi_{vv} \text{ (degrees)} \]

(a) \( p(\phi_c) \) for various values of \( \phi_0 \)

(b) \( p(\phi_c) \) for various values of \( \beta \)

**Figure 5-42:** Plots of the pdf \( p(\phi_c) \) of the co-pol phase difference \( \phi_c \) for various combinations of \( \phi_0 \) and \( \beta \).
Probability density (degree$^{-1}$)

\[ \phi_c = \phi_{hh} - \phi_{vv} \] (degrees)

**Figure 5-43:** Comparison of the measured histogram of $\phi_c$ extracted from a 1.2 GHz SAR image with $p(\phi_c)$ given by Eq. (5.157).
\[ \phi_c = \phi_{hh} - \phi_{vv} \text{ (degrees)} \]

**Field 19 (corn)**
- \( \theta = 19^\circ \)
- \( N = 523 \)
- Median = 8.4
- Mean = 9.0
- St. dev. = 35.5

**Field 50 (corn)**
- \( \theta = 50^\circ \)
- \( N = 705 \)
- Median = 110.5
- Mean = 109.5
- St. dev. = 63.4

**Figure 5-44:** Measured histograms of \( \phi_c \) for two cornfields, extracted from a 1.2 GHz SAR image.
Figure 5-45: Co-pol responses for (a) a metal sphere and (b) three areas in the San Francisco image.
Figure 5-46: Radar images of San Francisco showing the three measures of scattering randomness: (a) entropy scaled from 0 (black) to 1 (white); (b) pedestal height scaled from 0 (black) to 0.5 (white); and (c) the radar vegetation index scaled from 0 (black) to 1 (white).
Figure 5-47: L-band total-power image of a portion of the Black Forest in Germany acquired with the NASA/JPL AIRSAR system in the summer of 1991. Radar illumination is from the top.
Figure 5-48: RVI images of the area shown in Fig. 5-47 at three different frequencies. The RVI is scaled from 0 (black) to 1 (white). Note that the L-band RVI in the forested area is higher than the C-band RVI, while the C-band RVI is higher than the others in the agricultural areas.
\[ z = 0 \]

\[ z = -h_c \]

\[ z = -(h_c + h_t) \]

**Figure 5-49:** Scattering mechanisms for a forest canopy.
Figure 5-50: Results of the Freeman-Durden decomposition for the Black Forest image at (a) C-band, (b) L-band, and (c) P-band. Surface, dihedral-corner, and volume scattering components are displayed in blue, red, and green colors, respectively.